# Analysis of Hysteresis Phenomenon in a Current Sensor

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**This paper proposes a method to calculate the hysteresis phenomenon in Hall-effect current sensor. This method is composed of a simple treatment for magnetic hysteresis, a magnetic circuit and an electric circuit. The magnetic characteristics are expressed by sigmoid functions expressed by the residual flux density, saturated flux density and coercive force, which are easily obtained by magnetic companies. The effect of magnetic hysteresis in the core on its output signal is clarified.**

***Index Terms*— hysteresis, current sensor, Hall-effect**

## I. INTRODUCTION

A HALL-EFFECT sensor is one of the most important sensors which varies its output voltage in response to a magnetic field. Hall-effect sensors can be utilized for contactless measurements of DC and/or AC current [1], [2]. As they are prone to saturation and temperature drift when they are used as open-loop sensors, closed-loop types are preferred when high accuracy is essential. Closed-loop type of the Hall-effect sensor feeds back an opposing current into a secondary coil wound on the magnetic core to zero the flux produced in the magnetic core. They offer a high linearity, a fast response, and a low temperature drift. As Hall-effect sensors are mounted in the air-gap of magnetic core, hysteresis phenomenon can be caused in the output signal and there is a possibility of an uncertainty of zero. However, this hysteresis phenomenon has not been studied yet.

This paper proposes a method to calculate the hysteresis phenomenon in Hall-effect current sensor. This method is composed of a simple treatment for hysteresis and a magnetic circuit method. There are several methods to treat the magnetic hysteresis, for example, the Preisach model [3] and the Jiles-Atherton model [4]. In this paper, the magnetic characteristics are expressed by a sigmoid function, which is expressed by the residual flux density, the saturated flux density and the coercive force, which are easily obtained by magnetic companies. The effect of hysteresis in the core on its output signal is clarified.

## II. CURRENT SENSOR MODELING

Fig. 1 shows a simple closed-loop type Hall-effect current sensor to be studied. A current $I_1$ to be detected produces the magnetic field. The produced flux is concentrated in the core and is basically proportional to $I_1$. A Hall-element produces a voltage proportional to the flux density in the air-gap. The Hall element voltage is fed to an operational amplifier and to a push-pull amplifier. The push-pull amplifier drives the secondary winding on the core to make the flux in the core zero. The operation of secondary coil eliminates the non-linearity of magnetic core. The secondary current $i_2$ is converted to a voltage using a resistor between the secondary coil and the ground. A proper resistance value is selected for applications.

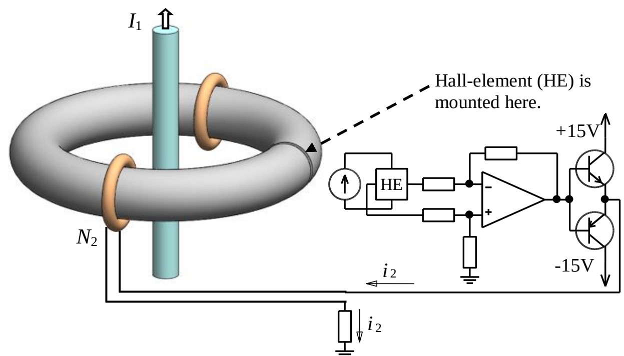


Fig. 1. Functional model for closed-loop Hall-effect current sensor.

The Ampere's circuit law is expressed by

$$I_1(t) + N_2 i_2(t) = H(t) l_g + H(t) l_i \tag{1}$$

where, $N_2$, $l_g$, and $l_i$ are the number of secondary coil, gap length, and core length, respectively. The voltage equations in secondary coils is given by

$$v_2(t) = R_2 i_2(t) + N_2 S \frac{dB(t)}{dt} \tag{2}$$

where, $R_2$, and $S$ are the resistance of secondary coil, and the area of core, respectively. If a proportional control is utilized

$$v_2(t) = -KB(t) \tag{3}$$

where, $K$ is a proportional gain. The magnetic hysteresis can be expressed by

$$H(t) = \frac{dH}{dB}(t)\{B(t) - B_0(t)\} + H_0(t) \tag{4}$$

Discretization with the 0-th order hold gives

$$B(n+1) = e^{A(n)T} B(n) + \frac{u(n)}{A(n)}\left\{e^{A(n)T} - 1\right\}$$

$$A(n) = \frac{-\dfrac{N_2 K}{R_2} - \dfrac{l_g}{\mu_0} - \dfrac{l_i}{\Delta\mu(n)}}{\dfrac{N_2{}^2 S}{R_2}} \tag{5}$$

$$u(n) = \frac{I_1(n) - H_0(n) l_i + \dfrac{l_i}{\Delta\mu(n)} B_0(n)}{\dfrac{N_2{}^2 S}{R_2}}$$

$$i_2(n+1) = -\frac{N_2 S}{R_2}\frac{B(n+1) - B(n)}{T} - \frac{K}{R_2} B(n+1) \tag{6}$$

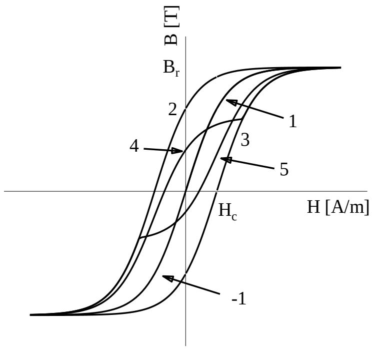


Fig. 2 Major and minor loops in B-H characteristic

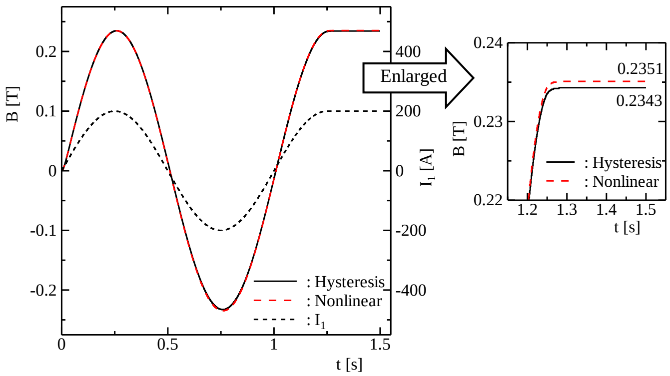


Fig. 3 Calculated waveform of magnetic flux density $B$ when no feedback loop.

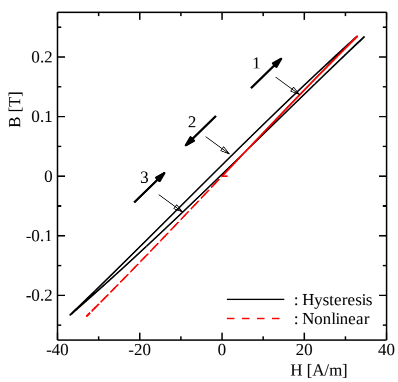


Fig. 4 B-H trajectory when no feedback loop.

where, $T$ and $\Delta\mu$ are a calculation time step and $dB/dH$, respectively.

This paper expresses the major and minor loops by sigmoid functions as shown in Fig. 2,

$$B = 2B_{\max}\left\{\left(\frac{1}{1+e^{-\alpha(H\pm Hc)}} - 0.5 \pm 0.5\right) fac \mp 0.5\right\} \qquad (7)$$

$$\alpha = -\frac{1}{H_c}\log\left\{\frac{2}{1+B_r/B_{\max}} - 1\right\} \qquad (8)$$

where, $B_{max}$, $B_r$, $H_c$, and $fac$ are the saturated flux density, the residual flux density, the coercive force, and a coefficient. The upper and lower signs denote the descending loop and ascending loop, respectively. The major loop is expressed when $fac$ = 1, and the initial loop is expressed when $H_c$= 0 and $fac$ = 1. When the magnetic flux density is changed from increase to decrease, the B-H loop is changed from an ascending one to a descending one, and vice versa. In this paper, $N_2$ = 1000, $l_i$ = 62.4 mm, $l_g$ = 1 mm, $S$ = 77 mm$^2$, $R_2$ = 10 Ω, and $T$ = 0.1ms.

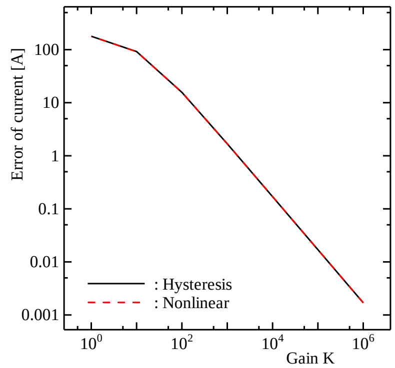


Fig. 5 Calculated error of current when changed the feedback gain $K$.

## III. Calculated Results

Fig.3 shows the waveform of flux density $B$ when the feedback loop is not included, and the current $I_1$ is input as shown in this figure. Here, nonlinear means that the B-H characteristic is expressed by the initial loop only. Although the flux density at the Hall-element is almost proportional to the current $I_1$, the flux densities are a little different. The relative difference is 0.2343/0.2351 = 0.9966, that is, 0.34 %. Fig.4 shows the B-H trajectory in this situation. First, the flux density increases on the initial loop, which is the same as nonlinear case, then decreases on a descending loop, and then increases on an ascending loop as shown in this figure. As a result the flux density differs from the nonlinear case. Fig. 5 shows the calculated current error of the closed-loop type of Hall-effect sensor, that is, $I_1 - N_2 I_2$. It is found that the error is decreases proportionally to the feedback gain $K$, when $K$ is large.

## IV. Conclusions

This paper has proposed a method to calculate the hysteresis phenomenon in Hall-effect current sensor. The proposed method has clarified that the closed-loop type of Hall-effect sensor offers the low magnetic hysteresis effect as well as the high linearity.

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